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Multi-channel Bethe-Salpeter equation*

Jaroslaw Boguszyński

State Office for Insurance Supervision, Warsaw, Poland
E-mail: jaroslaw.boguszynski@fuw.edu.pl

Hans Dieter Dahmen and Ralph Kretschmer

Siegen University, Fachbereich Physik, Siegen, Germany
E-mail: dahmen@physik.uni-siegen.de,
kretschm@hepth2.physik.uni-siegen.de

Leszek Łukaszuk

Soltan Institute for Nuclear Studies, Warsaw, Poland
E-mail: leszek.lukaszuk@fuw.edu.pl

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Abstract: A general form of multi-channel Bethe-Salpeter equation is considered. In contradistinction to the hitherto applied approaches, our coupled system of equations leads to the simultaneous solutions for all relativistic four-point Green functions (elastic and inelastic) appearing in a given theory. A set of relations which may be helpful in approximate treatments is given. An example of extracting useful information from the equations is discussed: we consider the most general trilinear coupling of N different scalar fields and obtain — in the ladder approximation — closed expressions for the Regge trajectories and their couplings to different channels in the vicinity of $\ell = -1$. Sum rules and an example containing non-obvious symmetry are discussed.

Keywords: Phenomenological Models, Sum Rules, Integrable Equations in Physics, Nonperturbative Effects.

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Contents

1. Introduction

The Bethe-Salpeter (BS) equation [1] has been widely applied in studies of two-body scattering and bound-state problems (reviews can be found in [2]). It is still the most important tool for describing bound states in quantum field theories. Recent applications include studies of quark-antiquark bound states by a combination of the BS equation and the Dyson-Schwinger equations in four-dimensional euclidean space [3, 4]. In addition, three-dimensional reductions of the BS equation are the basis of models with effective potentials that also lead to surprisingly good predictions [5]. Studies of the BS equation in four-dimensional Minkowski space on the basis of the perturbation theory integral representation [6], which avoid the difficulties of performing a Wick rotation [7], or attempts go beyond the ladder approximation [8] have lead to interesting results.

In some of the recent applications of the BS equation to scattering processes, the importance of a consistent treatment of coupled channels has been emphasized. Oller et al. [9] and Oset et al. [10] have developed a method that combines a multi-channel BS equation with chiral perturbation theory (ChPT) [11] to describe scalar mesons [9] and meson-nucleon scattering [10]. Here, the interest in the interpretation of scalar resonances stems from the possibility that some of these resonances may be glueballs or at least mixtures of $q\bar{q}$ and gluonic states [12]. In this context one should also mention the works of Janssen et al. [13] and Kamiński et al. [14], who employ coupled-channel versions of the Lippmann-Schwinger equation. For the description of meson-nucleon scattering, the inclusion of additional channels widens the range of applicability of the lowest order chiral lagrangian [15, 10].

Although these applications of the BS equation are phenomenologically quite successful, none of them provides a fully relativistic description. In a one-channel context, fully relativistic calculations were carried out for meson-meson scattering in [16] and for meson-nucleon scattering in [17].

But the BS equation is also a valuable tool in axiomatic QFT: it has been demonstrated ([18, 19] and references therein) that the exact Bethe-Salpeter formalism in the complex momentum space — together with its analytical dependence on both complex energy and complex angular momentum — follows from the basic principles of massive QFT such as locality, Lorentz covariance and the spectral condition. The possibility of an extension of this rigorous approach to theories of QCD type with a discrete spectrum of composite particles appearing as Regge-type particles has been recently announced [19].

Having in mind these impressive efforts and developments it seems rather surprising that the general form of the BS equation for two-body multi-channel problems has not been discussed in the literature. We find it therefore appropriate to start such a discussion.

Our contribution to this subject will be an investigation of some general properties of the solutions, leading to decomposition formulas for the case when the Bethe-Salpeter kernel and/or the Green functions can be subdivided into simpler parts. As an illustration of the usefulness of the multi-channel formulation, we then calculate the positions and couplings of Regge poles in the vicinity of $\ell = -1$ in a theory with general trilinear coupling of N different scalar fields in the ladder approximation. The treatment of this example is motivated not only by the fact that scalar theories of this kind, e.g. the Wick-Cutkosky model or the fully massive $\varphi^2\sigma$ model are widely used [7, 8], but also because studies of the $\ell = -1$ singularity in φ^3 theory often serve as a starting point for investigations of the diffractive region in QCD (see e.g. [20]).

As a result we obtain some novel features of Regge behaviour, in particular we derive non-trivial sum rules for the scattering amplitudes in the different channels.

The multi-channel approach presented here may be helpful to solve the important problem of understanding multi-channel hadron-hadron scattering as a consequence of QCD. There exist many approaches which could be a good point of departure for multi-channel BS predictions. What we have in mind are successful theories like ChPT [11], where effective actions with composite, hadronic degrees of freedom are derived, e.g. [21, 22]. Among these, the Global Colour Model described in [22, 4] respects the ultraviolet behaviour of QCD so that natural cutoffs appear in the effective hadronic couplings. In this context our example of a super-renormalizable theory with many channels involved may exhibit essential features of more realistic, future applications.

2. The symmetric and reduced forms of the Bethe-Salpeter equation

Consider the non-truncated Bethe-Salpeter Green function $\tau_{\rm BS}$:

$$\begin{aligned}\tau_{\rm BS}(a,x_1;b,x_2|a',x_1';b';x_2') &= \langle 0|\mathrm{T}[\varphi_a(x_1)\varphi_b(x_2)\varphi_{a'}(x_1')\varphi_{b'}(x_2')]|0\rangle - \\ &- \langle 0|\mathrm{T}[\varphi_a(x_1)\varphi_b(x_2)]|0\rangle\langle 0|\mathrm{T}[\varphi_{a'}(x_1')\varphi_{b'}(x_2')]|0\rangle\,.\end{aligned} \tag{2.1}$$

Here the fields $\varphi_a(x)$ may be bosons, fermions or ghosts — formally, we use one multicomponent field and the indices a contain the information on its boson, fermion or ghost nature. The function $\tau_{\rm BS}$ has the following symmetries:

$$\tau_{\rm BS}(a,x_1;b,x_2|a',x_1';b',x_2') = \delta^{\rm P}_{ab}\,\tau_{\rm BS}(b,x_2;a,x_1|a',x_1';b',x_2') \tag{2.2}$$

$$= \delta^{\rm P}_{a'b'}\,\tau_{\rm BS}(a,x_1;b,x_2|b',x_2';a',x_1')\,, \tag{2.3}$$

$$\tau_{\rm BS}(a,x_1;b,x_2|a',x_1';b',x_2') = \delta^{\rm P}_{aa'}\,\delta^{\rm P}_{ab'}\,\delta^{\rm P}_{ba'}\,\delta^{\rm P}_{bb'}\,\tau_{\rm BS}(a',x_1';b',x_2'|a,x_1;b,x_2)\,, \tag{2.4}$$

where $\delta^{\rm P}_{ab} = -1$ for φ_a, φ_b both being fermion (or ghost) fields, otherwise it is $+1$. The symmetry (2.4) results from the fact that the full propagator

$$\Delta'(a,x_1;b,x_2) = \delta^{\rm P}_{ab}\,\Delta'(b,x_2;a,x_1) = \langle 0|\mathrm{T}[\varphi_a(x_1)\varphi_b(x_2)]|0\rangle \tag{2.5}$$

in the subtracted term in (2.1) vanishes unless both φ_a and φ_b are either both fermions or bosons. But besides this, the propagator does not need to be diagonal in the discrete indices. In such a manner we admit both spinor propagators and the eventual mixing of different elementary fields. With (2.5) we can relate the matrix for the truncated, connected 4-point function

$$M(a,x_1;b,x_2|a',x_1';b',x_2') = \langle 0|\mathrm{T}[\varphi_a(x_1)\varphi_b(x_2)\varphi_{a'}(x_1')\varphi_{b'}(x_2')]|0\rangle|_{\rm trunc,\,conn} \tag{2.6}$$

to $\tau_{\rm BS}$ through

$$\tau_{\rm BS} = G + G\bullet M\bullet G\,, \tag{2.7}$$

where

$$\begin{aligned}G(a,x_1;b,x_2|a',x_1';b',x_2') &= \delta^{\rm P}_{ab}\,\Delta'(a,x_1;a',x_1')\Delta'(b,x_2;b',x_2') + \\ &+ \Delta'(a,x_1;b',x_2')\Delta'(b,x_2;a_1',x_1')\,,\end{aligned} \tag{2.8}$$

and the $\bullet$ product is defined as

$$\begin{aligned}(G\bullet M)(a,x_1;b,x_2|a',x_1';b',x_2') = \frac{1}{2}\sum_{c,d}\int \mathrm{d}^4y_1\,\mathrm{d}^4y_2\,G(a,x_1;b,x_2|c,y_1;d,y_2)\,\times \\ \times M(c,y_1;d,y_2|a',x_1';b',x_2')\,.\end{aligned} \tag{2.9}$$

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The factor 1/2 in (2.9) appears because G and M fulfill the same symmetry relations (2.2)–(2.4) as $\tau_{\rm BS}$ and hence all terms in the sum are counted twice. If the momentum-space Green functions are normalized like

$$M(a,k_1;b,k_2|a',k_1';b',k_2') = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_1'\,\mathrm{d}^4x_2'\,\mathrm{e}^{i(k_1x_1+k_2x_2-k_1'x_1'-k_2'x_2')} \times \\ \times M(a,x_1;b,x_2|a',x_1';b',x_2')\,, \tag{2.10}$$

then the matricial form (2.7) can also be used in 4-momentum space, with the symbol $\bullet$ denoting

$$\bullet = \frac{1}{2}\sum_{c,d}\int \frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{\mathrm{d}^4q_2}{(2\pi)^4}\,. \tag{2.11}$$

In momentum space, M contains an overall δ function,

$$iM(a,k_1;b,k_2|a',k_1';b',k_2') = (2\pi)^4\delta^4(k_1+k_2-k_1'-k_2') \times \\ \times \mathcal{M}(a,k_1;b,k_2|a',k_1';b',k_2')\,, \tag{2.12}$$

and if the transition matrix is defined by $S = 1 - iT$, then $\mathcal{M}$, taken at $k_1' + k_2' = k_1 + k_2$, denotes an off-shell extrapolation of the scattering amplitude for the process with outgoing particles a and b with momenta k_1 and k_2, respectively, and incoming particles $a'(k_1')$ and $b'(k_2')$. In the same way, G in momentum space contains δ functions:

$$G(a,k_1;b,k_2|a',k_1';b',k_2') = (2\pi)^8\left(\delta^{\rm P}_{ab}\Delta'(a;a',k_1)\Delta'(b;b',k_2)\delta^4(k_1-k_1')\delta^4(k_2-k_2') + \right. \\ \left. + \Delta'(a;b',k_1)\Delta'(b;a',k_2)\delta^4(k_1-k_2')\delta^4(k_2-k_1')\right) \tag{2.13}$$

with $\Delta'(a;b,k_1) = \int \mathrm{d}^4x\,\mathrm{e}^{ik_1x}\Delta'(a,x;b,0)$.

2.1 The symmetric form of the BS equation

The matrices $\tau_{\rm BS}$, G and M fulfill the symmetry relations (2.2)–(2.4). This leads to the symmetric form of the Bethe-Salpeter equation:

$$\tau_{\rm BS} = G + G\bullet B\bullet \tau_{\rm BS}\,, \tag{2.14}$$

and

$$M = B + B\bullet G\bullet M\,. \tag{2.15}$$

In these equations, B is the two-particle irreducible (for any two particles involved) part of M and also possesses the symmetries (2.2)–(2.4). The symmetry (2.4) leads to the alternative forms

$$\tau_{\rm BS} = G + \tau_{\rm BS}\bullet B\bullet G \qquad \text{and} \qquad M = B + M\bullet G\bullet B \tag{2.16}$$

of the BS equation. If we symbolically denote the solution of (2.15) by $M = M\{B,G\}$, then the solution of (2.14) can be written analogously:

$$M = M\{B,G\}\,, \qquad \tau_{\rm BS} = M\{G,B\}\,. \tag{2.17}$$

This will be useful for the discussion of the general relations.

2.2 The reduced form of the BS equation

In applications one may encounter forms of the BS equation that differ from the symmetric ones (2.14) and (2.15). These are special cases of a reduced form of the BS equation, i.e. forms where the BS equation is simplified for the price of loosing the explicitly symmetric shape.

The general transition may be described as follows: let G^{r} and B^{r} be arbitrary but satisfying

$$\begin{aligned} G &= G^{\mathrm{r}}(a,x_1;b,x_2|a',x_1';b',x_2') + \delta^{\mathrm{P}}_{a'b'}\, G^{\mathrm{r}}(a,x_1;b,x_2|b',x_2';a',x_1')\,, \\ B &= B^{\mathrm{r}}(a,x_1;b,x_2|a',x_1';b',x_2') + \delta^{\mathrm{P}}_{a'b'}\, B^{\mathrm{r}}(a,x_1;b,x_2|b',x_2';a',x_1')\,. \end{aligned} \tag{2.18}$$

Then (2.14) and (2.15) can be transformed into

$$\tau_{\mathrm{BS}} = G + G^{\mathrm{r}} \circ B^{\mathrm{r}} \circ \tau_{\mathrm{BS}}\,, \qquad M = B + B^{\mathrm{r}} \circ G^{\mathrm{r}} \circ M \tag{2.19}$$

and similarly for (2.16), with the $\circ$ product now being defined as

$$\circ = \sum_{c,d} \int \mathrm{d}^4 y_1\, \mathrm{d}^4 y_2\,, \tag{2.20}$$

i.e. without the factor 1/2 of (2.9). The forms (2.19) do not fit into the general relations (2.14) and (2.15). However, looking into the iteration procedure, we find

$$\tau_{\mathrm{BS}} = \frac{1}{1 - G^{\mathrm{r}} \circ B^{\mathrm{r}}} \circ G = \tau^{\mathrm{r}}_{\mathrm{BS}} + \delta^{\mathrm{P}}_{a'b'}\tau^{\mathrm{r}}_{\mathrm{BS}}\left((a',x_1') \leftrightarrow (b',x_2')\right) \tag{2.21}$$

with

$$\tau^{\mathrm{r}}_{\mathrm{BS}} = \frac{1}{1 - G^{\mathrm{r}} \circ B^{\mathrm{r}}} \circ G^{\mathrm{r}}\,. \tag{2.22}$$

Thus, we can deal with the equation

$$\tau^{\mathrm{r}}_{\mathrm{BS}} = G^{\mathrm{r}} + G^{\mathrm{r}} \circ B^{\mathrm{r}} \circ \tau^{\mathrm{r}}_{\mathrm{BS}}\,. \tag{2.23}$$

Similarly, one can decompose $M = M^{\mathrm{r}} + \delta^{\mathrm{P}}_{a'b'} M^{\mathrm{r}}((a',x_1') \leftrightarrow (b',x_2'))$ to work with

$$M^{\mathrm{r}} = \frac{1}{1 - B^{\mathrm{r}} \circ G^{\mathrm{r}}} \circ B^{\mathrm{r}} = B^{\mathrm{r}} + B^{\mathrm{r}} \circ G^{\mathrm{r}} \circ M^{\mathrm{r}}\,. \tag{2.24}$$

The functions G^{r} and B^{r} are not uniquely determined. Let us add that in typical ladder approximations the reduced form of the BS equation is in fact used. In these cases the choice of suitable G^{r} and B^{r} has been always implicitly made.

3. General relations

The basic principles of QFT ensure [18, 19] that the scattering matrix function M in the physical region can be obtained from the solution of eq. (2.15) in the euclidean region by analytic continuation in the invariant variables. Thus, if we had complete knowledge of the matrix functions B and G in the euclidean region, and if we would be able to exactly solve eq. (2.15) in this region, we could obtain complete information on M in the physical region. Generally none of these conditions is met and one has to rely on soundness of chosen scheme of approximations. Having this in mind we shall write below a few general relations which in our opinion may be helpful at least as a consistency check of approximations. We will throughout this section use the notation (2.14) and (2.15) for the symmetric form of the BS equation, but in view of (2.23) and (2.24) these relations can of course also be used for the reduced forms.

To begin with, consider the solution of (2.15) for given B, G:

$$M\{B,G\} = \frac{1}{1 - B \bullet G} \bullet B = B \bullet \frac{1}{1 - G \bullet B} . \tag{3.1}$$

This solution, treated as a functional of B and G has the following interesting property: for arbitrary G_1 one has

$$M\{B,G\} = M\left\{M\{B, G - G_1\}, G_1\right\} . \tag{3.2}$$

This equality has been found useful for the one-channel problem long time ago [23]. In fact eq. (3.2) comes as an answer to the following question: what is the relation between pairs (B,G) and (B_1,G_1) if both of them lead to the same M:

$$M\{B,G\} = M\{B_1, G_1\} . \tag{3.3}$$

Using eqs. (3.1) and (3.3) we get $B_1 = B + B \bullet [G - G_1] \bullet B_1$, i.e. from (2.15) we have $B_1 = M\{B, G - G_1\}$; inserting it into (3.3) we end up with relation (3.2).

Equation (3.2) leads to the following formula for $G = G_1 + G_2 + G_3 + \cdots + G_n$:

$$M\{B,G\} = M\left\{\ldots M\{M\{M\{B,G_1\},G_2\},G_3\},\ldots,G_n\right\} . \tag{3.4}$$

It is natural to ask whether similar formula holds in a situation when we subdivide B instead of G. The answer is yes, once we notice relation (2.7) between the off-shell scattering amplitude $M\{B,G\}$ and the non-truncated 4-point Green function $\tau_{\mathrm{BS}} = M\{G,B\}$:

$$M\{B,G\} = B + B \bullet M\{G,B\} \bullet B , \qquad M\{G,B\} = G + G \bullet M\{B,G\} \bullet G . \tag{3.5}$$

Inserting (3.4) with $G \leftrightarrow B$ in r.h.s. of (3.5) we get for $B = \sum_{k=1}^{n} B_k$

$$M\{B,G\} = B + B \bullet M\left\{\ldots M\{M\{M\{G,B_1\},B_2\},B_3\},\ldots,B_n\right\} \bullet B . \tag{3.6}$$

The formulae (3.4) and (3.6) may provide useful schemes of approximation for subdivision of Green functions and irreducible Bethe-Salpeter kernels. Of course in principle these two schemes can be combined. Let us mention that approaches with suitable divisions of G and/or B have already been useful both in the case of QED applications [24] and for deriving rigorous results concerning threshold behaviour in QFT [25]. The essential relation found in [25] exhibits convenience of using half-truncated amplitudes and kernels B_H, M_H:

$$M_H = M\{B, G\} \bullet G \,, \qquad B_H = B \bullet G \,. \tag{3.7}$$

For this case formula (2.15) yields $M_H = M\{B_H, 1\}$. Moreover, from eq. (3.5)

$$M\{1, B_H\} = 1 + M\{B_H, 1\} \,. \tag{3.8}$$

Using division of B_H in $M\{1, B_H\}$ together with eqs. (3.2) and (3.8) we recover in our notation Bros-Iagolnitzer relation (compare text before [25, eq. (5)]):

$$1 + M \bullet G = (1 + M'' \bullet G) \bullet (1 - B' \bullet G \bullet (1 + M'' \bullet G))^{-1} \,, \tag{3.9}$$

where $M = M\{B, G\}$, $M'' = M\{B - B', G\}$.

4. Arbitrary trilinear coupling scheme

We shall pass now to a problem where the use of the multi-channel approach itself can be clearly exhibited and discussed, namely to the example of a theory with general trilinear couplings of scalar fields. Models of this kind may have phenomenological applications when combined with effective, QCD-inspired couplings [11, 21, 22].

The most general trilinear coupling of N hermitian scalar fields φ_i $(i = 1, \ldots, N)$ is of the form:

$$\mathcal{L}_{\rm int}(x) = \frac{1}{3!} \sum_{i,j,k=1}^{N} c_{ijk} {:}\varphi_i(x) \varphi_j(x) \varphi_k(x){:} \tag{4.1}$$

The totally symmetric symbol c_{ijk} denotes the coupling constants.

Let us start with the off-shell amplitude:

$$iM^{\rm r}(i_2, q_2; i_4, q_4 | i_1, q_1; i_3, q_3) = (2\pi)^4 \delta^4(q_1 + q_3 - q_2 - q_4) \mathcal{M}^{i_1 i_2}_{i_3 i_4}(P_{\rm tot}, q_{13}, q_{24}) \tag{4.2}$$

with $P_{\rm tot} = q_1 + q_3 = q_2 + q_4$ and $q_{ij} = \frac{q_i - q_j}{2}$.

In the reduced form (2.24) of the Bethe-Salpeter equation in the ladder approximation, the matrix $G^{\rm r}$ can be chosen to be

$$G^{\rm r}(a, k_1; b, k_2 | a', k_1'; b', k_2') = (2\pi)^8 \delta_{aa'} \delta_{bb'} \Delta_a(k_1) \Delta_b(k_2) \delta^4(k_1 - k_1') \delta^4(k_2 - k_2') \tag{4.3}$$

with free propagators $\Delta_a(k) = i/[k^2 - m_a^2 + i\varepsilon]$, cf. (2.13). The Bethe-Salpeter equation for $\mathcal{M}^{i_1 i_2}_{i_3 i_4}(P_{\rm tot}, q_{13}, q_{24})$ then reads

$$\mathcal{M}^{i_1 i_2}_{i_3 i_4}(P_{\rm tot}, q_{13}, q_{24}) = \mathcal{B}^{i_1 i_2}_{i_3 i_4}(q_{13}, q_{24}) + \sum_{j,k} \int \mathrm{d}^4 q \, \mathcal{B}^{i_1 j}_{i_3 k}(q_{13}, q) \mathcal{G}^j_k(P_{\rm tot}, q) \times \\ \times \mathcal{M}^{j i_2}_{k i_4}(P_{\rm tot}, q, q_{24}) \,, \tag{4.4}$$

where $\mathcal{B}^{i_1 j}_{i_3 k}(q_{13}, q) = \sum_i c_{i_1 i j} c_{i_3 i k}/[(q_{13}-q)^2 - m_i^2 + i\varepsilon]$ is the Born term, and $\mathcal{G}^j_k(P_{\rm tot}, q)$ corresponds to diagonal part of (4.3):

$$\begin{aligned}\mathcal{G}^j_k(P_{\rm tot}, q) &= -\frac{i}{(2\pi)^4} \Delta_j\left(\frac{P_{\rm tot}}{2} + q\right) \Delta_k\left(\frac{P_{\rm tot}}{2} - q\right) \\ &= \frac{i}{(2\pi)^4} \frac{1}{(P_{\rm tot}/2 + q)^2 - m_j^2 + i\varepsilon} \frac{1}{(P_{\rm tot}/2 - q)^2 - m_k^2 + i\varepsilon} \,.\end{aligned} \tag{4.5}$$

In the c.m. frame ($P_{\rm tot} = (W, 0)$) the partial-wave decomposition of $\mathcal{M}^{i_1 i_2}_{i_3 i_4}$ is given by

$$\mathcal{M}^{i_1 i_2}_{i_3 i_4}(P^{\rm c.m.}_{\rm tot}, q_{13}, q_{24}) = \frac{1}{4\pi |\vec{q}_{13}||\vec{q}_{24}|} \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\vartheta) \times \\ \times \mathcal{M}_\ell^{i_1, i_3; i_2, i_4}(W, q^0_{13}, |\vec{q}_{13}|, q^0_{24}, |\vec{q}_{24}|) \,, \tag{4.6}$$

where ϑ is the angle between $\vec{q}_{13}$ and $\vec{q}_{24}$.

In what follows we shall simplify the notation, introducing an index $I(r,s)$ characterizing the channel (r,s), i.e.

$$(r,s) \longrightarrow I(r,s) = 1, \ldots, N^2 \,, \tag{4.7}$$

and define matrices $\mathsf{M}_{\ell IJ}$, $\mathsf{B}_{\ell IJ}$ and G_{IJ} by

$$\mathsf{M}_{\ell I(i,j) I(k,l)}(W, q^0_{13}, |\vec{q}_{13}|, q^0_{24}, |\vec{q}_{24}|) = \mathcal{M}_\ell^{i,j;k,l}(W, q^0_{13}, |\vec{q}_{13}|, q^0_{24}, |\vec{q}_{24}|) \,, \tag{4.8}$$

$$\mathsf{B}_{\ell I(i,j) I(k,l)}(q^0_{13}, |\vec{q}_{13}|, q^0_{24}, |\vec{q}_{24}|) = -2\pi \sum_{r=1}^{N} c_{irk} c_{jrl} Q_\ell(\beta_r(q^0_{13}, |\vec{q}_{13}|, q^0_{24}, |\vec{q}_{24}|)) \,, \tag{4.9}$$

$$\mathsf{G}_{I(i,j) I(k,l)}(W, k^0, |\vec{k}|, q^0, |\vec{q}|) = \delta_{I(i,j) I(k,l)} \delta(k^0 - q^0) \delta(|\vec{k}| - |\vec{q}|) \mathcal{G}^i_j(P_{\rm tot}, q) \,. \tag{4.10}$$

In (4.9) Q_ℓ is an associated Legendre function of the second kind, and its variable β_r is

$$\beta_r(q^0_{13}, |\vec{q}_{13}|, q^0_{24}, |\vec{q}_{24}|) = \frac{-(q^0_{13} - q^0_{24})^2 + \vec{q}^{\,2}_{13} + \vec{q}^{\,2}_{24} + m_r^2 - i\varepsilon}{2|\vec{q}_{13}||\vec{q}_{24}|} \,. \tag{4.11}$$

If the $\circ$ product for two such matrices is defined explicitly by

$$(\mathsf{X} \circ \mathsf{Y})_{IJ}(k^0, |\vec{k}|, q^0, |\vec{q}|) := \sum_{K=1}^{N^2} \int \mathrm{d}q'^0 \, \mathrm{d}|\vec{q}\,'| \, \mathsf{X}_{IK}(k^0, |\vec{k}|, q'^0, |\vec{q}\,'|) \times \\ \times \mathsf{Y}_{KJ}(q'^0, |\vec{q}\,'|, q^0, |\vec{q}|) \,, \tag{4.12}$$

then the Bethe-Salpeter equation for the partial wave amplitudes is again of the form (2.24),

$$\mathsf{M}_\ell = \mathsf{B}_\ell + \mathsf{B}_\ell \circ \mathsf{G} \circ \mathsf{M}_\ell \,, \tag{4.13}$$

with the solution

$$\mathsf{M}_\ell = \mathsf{B}_\ell \circ \frac{1}{1 - \mathsf{G} \circ \mathsf{B}_\ell} = \mathsf{M}\{\mathsf{B}_\ell, \mathsf{G}\} \,. \tag{4.14}$$

Here, one may split B_ℓ according to (4.9) into terms corresponding to the exchange of particles of type r,

$$\mathsf{B}_\ell = \sum_{r=1}^{N} \mathsf{B}_\ell^{(r)} \,, \tag{4.15}$$

$\mathsf{B}^{(r)}_{\ell I(i,j) I(k,l)} = -2\pi c_{irk} c_{jrl} Q_\ell(\beta_r)$. In this way we are naturally led to a decomposition of the form (3.6). Thus, M_ℓ can be written as

$$\mathsf{M}_\ell = \mathsf{B}_\ell + \mathsf{B}_\ell \circ \mathsf{M}\left\{ \ldots \mathsf{M}\{\mathsf{M}\{\mathsf{G}, \mathsf{B}_\ell^{(1)}\}, \mathsf{B}_\ell^{(2)}\}, \ldots, \mathsf{B}_\ell^{(N)} \right\} \circ \mathsf{B}_\ell \,. \tag{4.16}$$

This form of the solution may be useful for numerical studies of M_ℓ because the contributions of the various exchanged particles are now disentangled in a systematic way.

We will in the following focus our interest on Regge trajectories near $\ell = -1$; such a choice seems sound once we recall that studies of this singularity in φ^3 theory (i.e. only $c_{111} \neq 0$ in eq. (4.1)) often were and still are a point of departure for studies of the diffractive region in QCD (see, e.g. [20]). In our discussion we shall not go beyond the approximation of leading logarithms which means that — in analogy with the one-channel case [26] — Q_ℓ can be replaced by its leading ℓ-plane singularity, $Q_\ell(\beta) \to 1/(\ell+1)$. In this case, the matrix B_ℓ becomes constant,

$$\begin{aligned} \mathsf{B}_{\ell I(i,j) I(k,l)}(k^0, |\vec{k}|, q^0, |\vec{q}|) \longrightarrow -\frac{2\pi}{\ell+1} \mathsf{C}_{I(i,j) I(k,l)} &= -\frac{2\pi}{\ell+1} \sum_{r=1}^{N} c_{irk} c_{jrl} \\ &= \text{const} \,, \end{aligned} \tag{4.17}$$

so that

$$\mathsf{M}_\ell = -2\pi \mathsf{C} \circ \frac{1}{\ell + 1 + 2\pi \mathsf{G} \circ \mathsf{C}} \,. \tag{4.18}$$

Since C does not depend on the variables that are integrated by the $\circ$ product, (4.18) simplifies to give a usual matrix equation: one finds

$$\mathsf{C} \circ \mathsf{G} \circ \mathsf{C} = -\frac{1}{2\pi} \mathsf{CFC} \tag{4.19}$$

and so on, with the diagonal matrix F given by

$$\mathsf{F}_{I(i,j) I(k,l)}(W) = -2\pi \delta_{I(i,j) I(k,l)} \int \mathrm{d}q^0 \, \mathrm{d}|\vec{q}| \, \mathcal{G}^i_j(P_{\mathrm{tot}}, q) \equiv \delta_{I(i,j) I(k,l)} F_{I(i,j)}(W) \,. \tag{4.20}$$

Therefore, the solution of the Bethe-Salpeter equation is

$$\mathsf{M}_\ell = -2\pi \mathsf{C} \frac{1}{\ell + 1 - \mathsf{FC}} \,. \tag{4.21}$$

Here everything is to be understood as a matrix relation. We will in the following study the high-energy behaviour of the amplitude in the crossed channel. To do this, we define the symmetric matrix $\hat{\mathsf{C}} = \sqrt{\mathsf{F}}\mathsf{C}\sqrt{\mathsf{F}}$ and write (4.21) in the form

$$\mathsf{M}_\ell = -2\pi \sqrt{\mathsf{F}}^{-1} \hat{\mathsf{C}} \frac{1}{\ell + 1 - \hat{\mathsf{C}}} \sqrt{\mathsf{F}}^{-1} \,. \tag{4.22}$$

We shall limit our considerations to $P_{\rm tot}^2 \leq 0$ so that F_{IJ} comes out to be real and positive. Since $\hat{\mathsf{C}}$ is symmetric, we can diagonalize it: let $\mathsf{v}^{[i]}$ denote the normalized eigenvectors,

$$\hat{\mathsf{C}}\mathsf{v}^{[i]} = \lambda^{[i]}\mathsf{v}^{[i]} \,, \qquad i = 1, \ldots, N^2 \,, \tag{4.23}$$

then

$$\mathsf{M}_\ell = -2\pi \sqrt{\mathsf{F}}^{-1} \left(\sum_i \frac{\mathsf{v}^{[i]}\lambda^{[i]}\mathsf{v}^{[i]\dagger}}{\ell + 1 - \lambda^{[i]}} \right) \sqrt{\mathsf{F}}^{-1} \,. \tag{4.24}$$

Thus, the Regge trajectories are given by

$$\alpha^{[i]} = \ell_{\rm pole} = \lambda^{[i]} - 1 \,. \tag{4.25}$$

With (4.24) we are ready to use the Mandelstam-Sommerfeld-Watson transformation [26, 27, 28] to obtain the high-energy amplitude in the crossed channel ($t = (q_1 + q_3)^2 = W^2$ fixed, $s = (q_1 - q_2)^2 \to \infty$), with the result

$$\begin{aligned} \mathsf{M}_{\rm poles} &\overset{s\longrightarrow\infty,}{\underset{t\ \text{fixed}}{=}} \frac{1}{s} \sqrt{\mathsf{F}}^{-1} \left(\sum_i \mathsf{v}^{[i]}\lambda^{[i]} s^{\lambda^{[i]}} \mathsf{v}^{[i]\dagger} \right) \sqrt{\mathsf{F}}^{-1} \\ &= \frac{1}{s} \sqrt{\mathsf{F}}^{-1} \left(\hat{\mathsf{C}} \exp(\hat{\mathsf{C}} \ln s) \right) \sqrt{\mathsf{F}}^{-1} \,. \end{aligned} \tag{4.26}$$

The states $\mathsf{v}^{[i]}$ are eigenstates of the hermitian matrix $\hat{\mathsf{C}}$, thus they are orthogonal. This is, however, no longer true for the states $\sqrt{\mathsf{F}}^{-1}\mathsf{v}^{[i]}$. As the dual base for these states we introduce the states

$$\mathsf{u}^{[i]} = \sqrt{\mathsf{F}}\mathsf{v}^{[i]} \qquad \text{with} \qquad \mathsf{u}^{[i]\dagger}\mathsf{F}^{-1}\mathsf{u}^{[j]} = \delta_{ij} \,. \tag{4.27}$$

The only matrix elements of $\mathsf{M} = \mathsf{M}_{\rm poles}$ in this base that are non zero are $\mathsf{u}^{[i]\dagger}\mathsf{M}\mathsf{u}^{[i]}$, $i = 1, \ldots, N^2$. The high-energy behaviour of each of these matrix elements is governed by only one trajectory,

$$\mathsf{u}^{[i]\dagger}\mathsf{M}\mathsf{u}^{[j]} = \delta_{ij}\lambda^{[i]} s^{\lambda^{[i]}-1} \,, \tag{4.28}$$

although the states $\mathsf{u}^{[i]}$ are in general not orthogonal. Equation (4.28) leads to the following sum rules for the scattering amplitudes:

$$\sum_{I,J=1}^{N^2} \mathsf{v}_I^{[i]} \mathsf{v}_J^{[j]} \sqrt{F_I F_J} \mathsf{M}_{IJ} = \delta_{ij} \lambda^{[i]} s^{\lambda^{[i]}-1} . \tag{4.29}$$

Note that some of these relations simplify whenever $\lambda^{[i]} = 0$. In this case,

$$\mathsf{M}\mathsf{u}^{[i]} = 0 , \qquad \text{for} \qquad \lambda^{[i]} = 0 \tag{4.30}$$

(or equivalently $\mathsf{C}\mathsf{u}^{[i]} = 0$), i.e. states corresponding to fixed Regge singularities are transparent in our approximation.

As an illustration, let us consider the interaction

$$\mathcal{L}_{\mathrm{I}} = \frac{g_1}{2!} :\varphi^2 \sigma: + \frac{g_2}{3!} :\sigma^3: , \tag{4.31}$$

which is a special case of (4.1) for $N = 2$, $\varphi_1 = \varphi$, $\varphi_2 = \sigma$, and $c_{112} = c_{121} = c_{211} = g_1$, $c_{222} = g_2$, $c_{111} = c_{122} = c_{212} = c_{221} = 0$. Let $I(1,1) = 1$, $I(2,2) = 2$, $I(1,2) = 3$, and $I(2,1) = 4$, then the matrix C is block diagonal due to the symmetry of the total lagrangian with respect to $\varphi \to -\varphi$,

$$\mathsf{C} = \begin{pmatrix} \mathsf{C}' & 0 \\ 0 & \mathsf{C}'' \end{pmatrix}, \qquad \mathsf{C}' = \begin{pmatrix} g_1^2 & g_1^2 \\ g_1^2 & g_2^2 \end{pmatrix}, \qquad \mathsf{C}'' = \begin{pmatrix} g_1 g_2 & g_1^2 \\ g_1^2 & g_1 g_2 \end{pmatrix}, \tag{4.32}$$

and F is diagonal with $F_4 = F_3$, so the matrix $\hat{\mathsf{C}}$ is

$$\hat{\mathsf{C}} = \begin{pmatrix} \hat{\mathsf{C}}' & 0 \\ 0 & \hat{\mathsf{C}}'' \end{pmatrix}, \qquad \hat{\mathsf{C}}' = \begin{pmatrix} F_1 g_1^2 & \sqrt{F_1 F_2} g_1^2 \\ \sqrt{F_1 F_2} g_1^2 & F_2 g_2^2 \end{pmatrix}, \qquad \hat{\mathsf{C}}'' = F_3 \mathsf{C}'' . \tag{4.33}$$

Diagonalizing $\hat{\mathsf{C}}'$, $\hat{\mathsf{C}}''$ we get eigenvalues

$$\lambda^{[1,2]} = \frac{1}{2} \left(F_1 g_1^2 + F_2 g_2^2 \pm \sqrt{(F_1 g_1^2 - F_2 g_2^2)^2 + 4 F_1 F_2 g_1^4} \right) \tag{4.34}$$

of $\hat{\mathsf{C}}'$ and

$$\lambda^{[3,4]} = F_3 g_1 (g_2 \pm g_1) \tag{4.35}$$

of $\hat{\mathsf{C}}''$. The corresponding eigenvectors are

$$\mathsf{v}^{[1]} = \frac{1}{\sqrt{d}} \begin{pmatrix} \sqrt{a} \\ \sqrt{b} \\ 0 \\ 0 \end{pmatrix}, \qquad \mathsf{v}^{[2]} = \frac{1}{\sqrt{d}} \begin{pmatrix} \sqrt{b} \\ -\sqrt{a} \\ 0 \\ 0 \end{pmatrix},$$

$$\mathsf{v}^{[3]} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \qquad \mathsf{v}^{[4]} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix},$$

with $d = \sqrt{(F_1 g_1^2 - F_2 g_2^2)^2 + 4F_1F_2 g_1^4}$, $a = (d + F_1 g_1^2 - F_2 g_2^2)/2$, and $b = (d - F_1 g_1^2 + F_2 g_2^2)/2$. Out of the ten relations described by (4.29), four are trivial due to the fact that M is block diagonal. The remaining relations read (using the symmetry of M)

$$\begin{aligned}
aF_1 \mathsf{M}_{\varphi,\varphi\to\varphi,\varphi} + 2F_1F_2 g_1^2 \mathsf{M}_{\varphi,\sigma\to\varphi,\sigma} + bF_2 \mathsf{M}_{\sigma,\sigma\to\sigma,\sigma} &= (\lambda^{[1]} - \lambda^{[2]})\lambda^{[1]} s^{\lambda^{[1]}-1}\,, \\
bF_1 \mathsf{M}_{\varphi,\varphi\to\varphi,\varphi} - 2F_1F_2 g_1^2 \mathsf{M}_{\varphi,\sigma\to\varphi,\sigma} + aF_2 \mathsf{M}_{\sigma,\sigma\to\sigma,\sigma} &= (\lambda^{[1]} - \lambda^{[2]})\lambda^{[2]} s^{\lambda^{[2]}-1}\,, \\
\mathsf{M}_{\varphi,\varphi\to\sigma,\sigma} + 2\mathsf{M}_{\varphi,\sigma\to\sigma,\varphi} + \mathsf{M}_{\sigma,\sigma\to\varphi,\varphi} &= 2g_1(g_1+g_2) s^{\lambda^{[3]}-1}\,, \\
\mathsf{M}_{\varphi,\varphi\to\sigma,\sigma} - 2\mathsf{M}_{\varphi,\sigma\to\sigma,\varphi} + \mathsf{M}_{\sigma,\sigma\to\varphi,\varphi} &= 2g_1(g_2-g_1) s^{\lambda^{[4]}-1}\,, \\
F_1 g_1^2 \mathsf{M}_{\varphi,\varphi\to\varphi,\varphi} + (F_2 g_2^2 - F_1 g_1^2)\mathsf{M}_{\varphi,\sigma\to\varphi,\sigma} - F_2 g_1^2 \mathsf{M}_{\sigma,\sigma\to\sigma,\sigma} &= 0\,, \\
\mathsf{M}_{\varphi,\varphi\to\sigma,\sigma} - \mathsf{M}_{\sigma,\sigma\to\varphi,\varphi} &= 0\,,
\end{aligned} \tag{4.36}$$

where, e.g. $\mathsf{M}_{\varphi,\sigma\to\varphi,\sigma}$ is short for $\mathcal{M}((\vec{p}_1,\varphi),(\vec{p}_2,\sigma)\to(\vec{p}_3,\varphi),(\vec{p}_4,\sigma))$.

Of course, these relations are only valid in the Regge limit $s\to\infty$, t fixed, and apply only to Regge behaviour, so that a vanishing combination of amplitudes means that this combination does not exhibit Regge behaviour.

In the example, eigenvalues $\lambda^{[i]}$ can vanish only for $g_2 = \pm g_1$. If, say, $g_2 = g_1$ (i.e. $\lambda^{[1]} = g_1^2(F_1+F_2)$, $\lambda^{[2]} = 0$, $\lambda^{[3]} = 2g_1^2 F_3$, $\lambda^{[4]} = 0$), the relations (4.36) simplify to

$$\begin{aligned}
\mathsf{M}_{\varphi,\varphi\to\varphi,\varphi} = \mathsf{M}_{\sigma,\sigma\to\sigma,\sigma} = \mathsf{M}_{\varphi,\sigma\to\varphi,\sigma} &= g_1^2 s^{\lambda^{[1]}-1}\,, \\
\mathsf{M}_{\varphi,\varphi\to\sigma,\sigma} = \mathsf{M}_{\sigma,\sigma\longrightarrow\varphi,\varphi} = \mathsf{M}_{\varphi,\sigma\longrightarrow\sigma,\varphi} &= g_1^2 s^{\lambda^{[3]}-1}\,.
\end{aligned} \tag{4.37}$$

If moreover, we assumed $m_\sigma = m_\varphi$, then $\lambda^{[1]} = \lambda^{[3]}$ and all the amplitudes standing on the l.h.s. of (4.37) are the same. This symmetry can be traced back to the fact that for $g_1 = g_2 \equiv g$ we can rewrite eq. (4.31) as a sum of two separate $\chi_\pm^3$ terms

$$\mathcal{L}_{\rm int}(g_1 = g_2 \equiv g) = \frac{g}{2}\frac{1}{3!}\left[:\chi_+^3: + :\chi_-^3:\right], \tag{4.38}$$

where $\chi_\pm = \sigma \pm \varphi$. Thus, in the special case $m_\sigma = m_\varphi$ the total lagrangian can be separated into two of identical form and hence the complete symmetry follows.

5. Concluding remarks

The general results of sections 2 and 3 provide a clear and general scheme of dealing with various approximations in the multi-channel Bethe-Salpeter equation. Moreover, the results derived in section 4 show that the multi-channel Bethe-Salpeter formalism is tractable and leads to non-trivial physical relations, e.g. sum rules for scattering amplitudes. In our opinion, future applications may include the treatment of the e^+e^- and $\gamma\gamma$ channels of parapositronium, and the Bethe-Salpeter analysis of hadronic multi-channel processes hitherto treated not fully relativistic. In the latter case theoretical input could be provided by effective couplings derived from QCD [11, 21, 22].

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